

Worked Example 1 (Version 3)

Design of cantilever pole retaining walls to resist earthquake loading for residential sites

Worked example to accompany MBIE Guidance on the seismic design of retaining structures for residential sites in Greater Christchurch (Version 2) November 2014

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Introduction

Cantilever timber pole walls are probably the most commonly used form of retaining wall for residential purposes. The poles may also be of steel section for more heavily loaded walls. The design of these walls is relatively straight forward but several modes of failure need to be considered. The most common problem with these walls is rotation about the base because of inadequate depth of embedment of the poles, often because of over-estimating the appropriate soil strength parameters or use of wrong design models.

1.1 Possible modes of failure

Possible modes of failure for cantilever pole retaining walls are illustrated in cartoon fashion in Figure X.1. A complete design should address each of these modes of failure where appropriate.

- a) **Foundation failure:** The embedded pole foundations rotate through the soil.
- b) **Pole structural failure:** The poles fail in bending. Most likely location is at the ground surface where the poles are embedded in substantial concrete foundations otherwise may be below the ground surface.
- c) **Sliding failure of wall:** Possible mode for non-cohesive soils. Wall moves outwards with passive failure of soil in front of wall and active failure of soil behind wall. Factor of safety controlled by increasing depth of embedment of wall. Unlikely to govern design for typical cases.
- d) **Deep seated rotational failure:** Possible mode for cohesive soils. Factor of safety controlled by increasing depth of embedment of wall. Factor of safety calculated using limiting equilibrium “Bishop” analysis or similar. Unlikely to govern unless wall is embedded into sloping ground with sloping backfill or there is a weak layer at the toe of the wall.

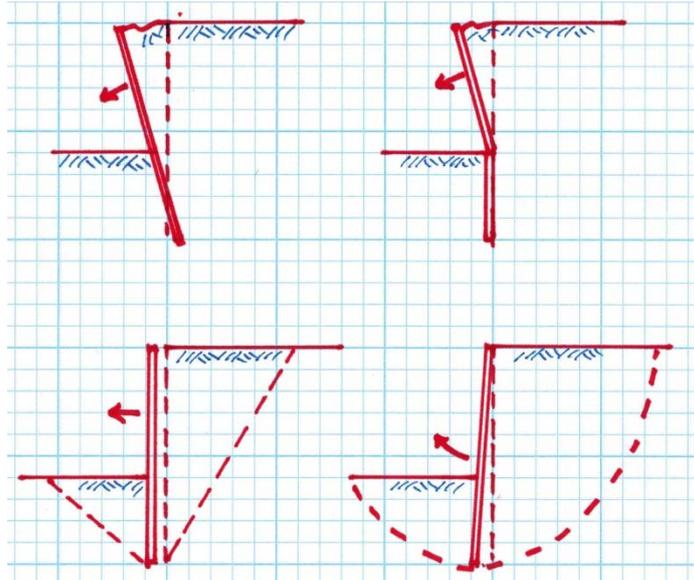


Figure X.1. Possible modes of failure for cantilever pole retaining walls.

Typically, the poles will be spaced relatively close together and the pole foundations should be treated as a continuous strip or bulkhead. The following simplified procedure treats the wall as a continuous bulkhead. If the poles are spaced apart more than 3 – 4 diameters (the diameter of the concrete encased pole foundation) then the foundations should be treated as individual laterally loaded “piles” using an appropriate design procedure (eg Broms, 1964).

This procedure is intended to be readily calculated by hand, although use of calculation software such as Mathcad or Excel will be useful for design iterations. The example calculations are made here using Mathcad.

1.2 Example Wall

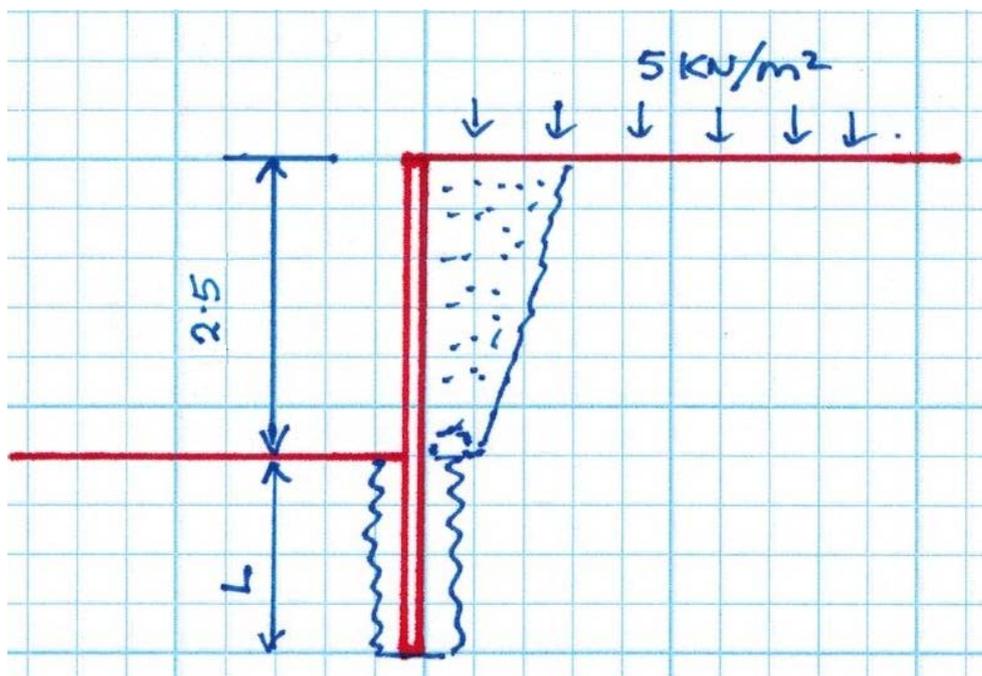


Figure X.2. Cantilever pole retaining wall example.

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The example wall is shown in Figure X.2. The wall is assumed to be located in the Christchurch Port Hills. The following design assumptions were made:

Soil type: Port Hills loess

Strength parameters: $c = 0$, $\phi = 30$ degrees

Drained strength parameters for Port Hills loess were assumed for the long term, gravity only load case. For the earthquake load case, the foundations in loess were designed assuming undrained strength, $c = 50 \text{ KN/m}^2$, $\phi = 0$ degrees, when calculating the passive resistance of the foundation soil. (Following the recommendations given in the Guidelines).

Wall situation: Case 3: Retaining wall supporting dwelling

Surcharge: The surcharge from the dwelling was assumed to be 5 KN/m^2 for the gravity case and 4 KN/m^2 for the earthquake case, averaged across the active soil wedge. Surcharge should be calculated using:

$$\omega = 1.2 G + 0.4 Q \text{ for the gravity case}$$

$$\omega = G + 0.3 Q \text{ for the earthquake case.}$$

Seismic parameters:

$$C(T) = C_h(T)ZRN(T, D) \quad \text{Equation (1.1) from Guidelines}$$

$$C_h(T) = 1.33 \text{ for Class C assuming shallow soil site}$$

$$Z = 0.3 \text{ for Christchurch for ULS}$$

$$R = \text{Return period factor} = 1.0 \text{ for Importance Level 2 walls, ULS}$$

$$N(T, D) = \text{Near fault factor which may be taken} = 1.0 \text{ for residential retaining walls}$$

$$C(T) = 0.3 \times 1.33 = 0.4$$

$$C(T, A_{topo}) = C(T)A_{topo} \quad \text{Equation (1.2) from Guidelines}$$

$$A_{topo} = 1.0 \text{ assuming site is not near cliff edge or ridge top}$$

$$C(T, A_{topo}) = 0.4 \times 1.0 = 0.4$$

$$k_h = C(T, A_{topo})W_d \quad \text{Equation (1.3) from Guidelines}$$

$$W_d = \text{wall displacement factor, given in Table 2 from Guidelines as 0.5 (refer to Table 1 for wall case then, Table 2 for } W_d)$$

$$k_h = 0.4 \times 0.5 = 0.2$$

Note that by adopting $W_d = 0.5$ it is implicitly assumed that the wall and the retained ground are likely to yield and accumulate permanent displacement as a result of the design earthquake. Wall elements including the poles and anchor tendons must be sufficiently resilient and/or ductile to accommodate the displacement. Some settlement of retained material behind the wall should also be expected following a seismic event.

Step 1. Initial trial geometry

For the example assume that the pole spacing will be at 1.5 m centres and that the poles will be inserted into 500 mm diameter holes and backfilled with concrete. At this spacing (3 diameters) it will be appropriate to treat the wall as a continuous bulkhead. Typically the pole spacing will be governed by the strength of the lagging.

Step 2. Wall overturning (gravity case)

Determine the depth of embedment of the poles by considering a simplified overturning analysis as shown in Figure X.3.

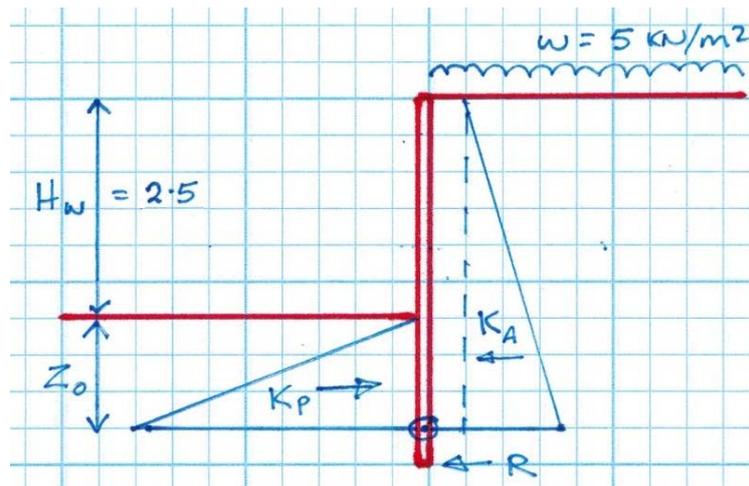


Figure X.3. Gross pressure method of analysis for embedded cantilever wall (gravity case).

The wall is assumed to rotate at a depth Z_0 below the ground surface, with active soil pressure behind the wall acting to overturn the wall about the point of rotation, and passive soil pressure in front of the wall acting to resist overturning.

The pole is assumed to have additional embedment below the point of rotation sufficient to provide the necessary reaction force, R .

The depth, Z_0 is increased by trial and error until the factor of safety against rotation about O is $FS = 1.5$ for the gravity case or $FS = 1.0$ for the earthquake case (refer to Table 4 in the Guidelines).

The depth of embedment is taken as $L = 1.2 Z_0$ to ensure that there is sufficient embedment to generate the necessary reaction at the toe of the pole.

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Pole Cantilever Wall Overturning

$$KN := 1000 \cdot N$$

$$H_w := 2.5 \cdot m \quad \text{Height of wall}$$

$$Z_o := 2.06 \cdot m \quad \text{Depth of rotation - by trial and error}$$

$$\gamma_a := 18 \cdot \frac{KN}{m^3} \quad \text{Soil unit weight}$$

$$K_{ah} := 0.29 \quad \text{From NAVFAC DM7 } \phi = 30 \text{ degrees, } \delta = 2/3 \phi$$

$$K_{ph} := 5.6 \quad \text{From NAVFAC DM7 } \phi = 30 \text{ degrees, } \delta = \phi$$

$$\omega := 5 \cdot \frac{KN}{m^2} \quad \text{surchage}$$

Driving moment from active soil pressure above point of rotation

$$M_d := K_{ah} \cdot \left[\frac{1}{2} \omega \cdot (H_w + Z_o)^2 + \frac{1}{6} \gamma_a \cdot (H_w + Z_o)^3 \right]$$

$$M_d = 97.568 \cdot KN \cdot \frac{m}{m}$$

Restoring moment about base from passive soil pressures

$$M_r := K_{ph} \cdot \gamma_a \cdot \frac{Z_o^3}{6} \quad M_r = 146.863 \cdot KN \cdot \frac{m}{m}$$

Check Factor of Safety

$$FS := \frac{M_r}{M_d} \quad FS = 1.505 \quad \text{OK}$$

Depth of embedment

$$L_{embed} := Z_o \cdot 1.2 \quad L_{embed} = 2.472 \text{ m}$$

Note that K_{ah} and K_{ph} refer to the horizontal components of the active and passive thrusts. The interface friction angle between the soil backfill and the wall δ was assumed to be equal to $2/3 \phi$ for calculating the active thrust and δ was assumed to be equal to ϕ for calculating the passive resistance for concrete encased poles poured in-situ.

Step 3. Wall overturning (earthquake case)

Check that the depth of embedment of the poles is adequate for the earthquake case by considering a simplified overturning analysis as shown in Figure X.4. For the earthquake case, the undrained shear strength of the foundation soil may be assumed for Port Hills loess when calculating the passive soil resistance, $S_u = 50 \text{ KN/m}^2$ was assumed for the example. The passive soil distribution is as shown in Figure X.4 with the cohesive contribution = $2c$, $c = S_u$, and $K_p = 1$ for $\phi = 0$. Where the ground surface immediately in front of the wall is exposed, the passive resistance may be ineffective near to the ground surface because of desiccation and cracking. For the example, the upper 0.5 m of passive

resistance was neglected. For other situations where the ground surface is protected by pavement it may be appropriate to include the passive soil resistance over the full depth of embedment, using judgement.

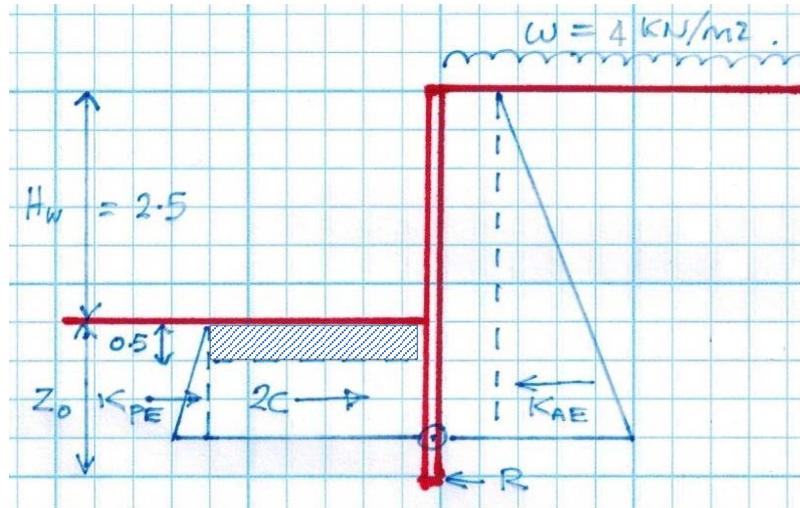


Figure X.4. Gross pressure method of analysis for embedded cantilever wall (earthquake case)

For the earthquake case, K_{ah} is replaced in the calculations by K_{aeh} , calculated using the Mononobe-Okabe equations, calculated as below.

Calculation of K_{aeh}

- $k_h := 0.2$ horizontal acceleration in g $\theta := \text{atan}(k_h) \quad \theta = 11.31\text{-deg}$
- $\beta := 0\text{-deg}$ slope of the back of the wall
- $i := 0\text{-deg}$ slope of the backfill
- $\phi := 30\text{-deg}$ angle of internal friction
- $\delta_a := 20\text{-deg}$ angle of interface friction (active)

Calculation

$$D := \left[1 + \left(\frac{\sin(\phi + \delta_a) \cdot \sin(\phi - \theta - i)}{\cos(\delta_a + \beta + \theta) \cos(i - \beta)} \right)^{0.5} \right]^2 \quad D = 2.359$$

$$K_{AE} := \frac{\cos(\phi - \theta - \beta)^2}{\cos(\theta) \cos(\beta)^2 \cos(\beta + \delta_a + \theta) \cdot D} \quad K_{AE} = 0.454$$

Equivalent Horizontal Component

$$K_{AEH} := \cos(\delta_a + \beta) K_{AE} \quad K_{AEH} = 0.427$$

Check of embedment depth

Pole Cantilever Wall Overturning (Cohesive foundations)

$$\begin{aligned} K_N &:= 1000 \cdot \text{N} \\ H_w &:= 2.5 \cdot \text{m} && \text{Height of wall} \\ Z_o &:= 2.06 \cdot \text{m} && \text{Depth of rotation - by trial an error} \\ \gamma_a &:= 18 \cdot \frac{\text{KN}}{\text{m}^3} && \text{Soil unit weight} \\ S_u &:= 50 \cdot \frac{\text{KN}}{\text{m}^2} && \text{Undrained shear strength of founding soil} \\ K_a &:= 0.427 && \text{From M-O equations} \\ K_p &:= 1 && \text{For } \phi = 0 \text{ soil} \\ \omega &:= 4 \cdot \frac{\text{KN}}{\text{m}^2} && \text{surcharge (factored)} \end{aligned}$$

Driving moment from active soil pressure above point of rotation

$$M_d := K_a \cdot \left[\frac{1}{2} \omega \cdot (H_w + Z_o)^2 + \frac{1}{6} \gamma_a \cdot (H_w + Z_o)^3 \right]$$
$$M_d = 139.221 \cdot \text{KN} \cdot \frac{\text{m}}{\text{m}}$$

Restoring moment about base from passive soil pressures

$$M_r := K_p \cdot \gamma_a \cdot \frac{Z_o^3}{6} + 2 \cdot S_u \cdot \frac{(Z_o - 0.5 \cdot \text{m})^2}{2} \quad M_r = 147.905 \cdot \text{KN} \cdot \frac{\text{m}}{\text{m}}$$

Check Factor of Safety

$$FS := \frac{M_r}{M_d} \quad FS = 1.062 \quad \text{OK} > 1.0$$

Depth of embedment

$$L_{\text{embed}} := Z_o \cdot 1.2 \quad L_{\text{embed}} = 2.472 \text{m}$$

For the example, it was found that the depth of embedment determined for the gravity load case was also suitable for the earthquake load case ($L = 2.5 \text{ m}$).

Step 4. Pole strength

The pole structural elements may fail in bending. For poles encased in concrete foundations bending failure is most likely to occur either at ground level, where the concrete encasement terminates, or below ground level at the depth of maximum bending moment if composite bending capacity of the concrete encased pole is considered. Both cases need to be checked. For poles embedded directly into soil, bending failure will occur below ground level at the depth of maximum bending moment.

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Where there is a substantial concrete slab or other restraint at ground level, then pole bending will be critical at the location of the restraint. Bearing of the timber pole against the slab should also be checked in such cases. Note that before such restraint may be assumed, it is necessary to establish a realistic load path for the necessary restraining forces.

The calculated soil pressure loads acting against the back of the wall should have load factors applied in accordance with NZS 1170.0:2002 (see Guidelines).

For the example, pole bending moments are calculated at ground level, where the concrete encasement terminates, and at the depth of maximum bending moment (zero shear). The calculations are made with reference to the soil pressure diagram in Figure X.3 for the gravity case and Figure X.4 for the earthquake case.

Pole Cantilever Wall Bending (gravity)

$$KN := 1000 \cdot N$$

$$H_w := 2.5 \cdot m \quad \text{Height of wall}$$

$$\gamma_a := 18 \cdot \frac{KN}{m^3} \quad \text{Soil unit weight}$$

$$K_{ah} := 0.29 \quad \text{From NAVFAC DM7 } \phi = 30 \text{ degrees, } \delta = 2/3 \phi$$

$$K_{ph} := 5.6 \quad \text{From NAVFAC DM7 } \phi = 30 \text{ degrees, } \delta = \phi$$

$$\omega := 5 \cdot \frac{KN}{m^2} \quad \text{surchage}$$

$$\alpha := 1.5 \quad \text{load factor, gravity}$$

$$L_s := 1.5 \cdot m \quad \text{pole spacing}$$

Bending moment in pole at ground level

$$M_{star} := \alpha \cdot K_{ah} \cdot \left(\frac{1}{2} \cdot \omega \cdot H_w^2 + \frac{1}{6} \cdot \gamma_a \cdot H_w^3 \right) \cdot L_s \quad M_{star} = 40.781 \cdot KN \cdot m$$

Depth below ground to zero shear (max bending moment)

$$z_{bmax} := 0.815 \cdot m \quad \text{set SF to zero by trial and error}$$

$$SF_{bmax} := K_{ah} \cdot \left[\omega \cdot (H_w + z_{bmax}) + \frac{1}{2} \cdot \gamma_a \cdot (H_w + z_{bmax})^2 \right] \cdot L_s - K_{ph} \cdot \frac{1}{2} \cdot \gamma_a \cdot z_{bmax}^2 \cdot L_s$$

$$SF_{bmax} = 0.018 \cdot KN$$

Maximum bending moment below ground

$$M_{star2} := \alpha \cdot \left[K_{ah} \cdot \left[\frac{1}{2} \cdot \omega \cdot (H_w + z_{bmax})^2 + \frac{1}{6} \cdot \gamma_a \cdot (H_w + z_{bmax})^3 \right] - K_{ph} \cdot \left(\frac{1}{6} \cdot \gamma_a \cdot z_{bmax}^3 \right) \right] \cdot L_s$$

$$M_{star2} = 68.774 \cdot KN \cdot m$$

Note that the dependable bending capacity of the timber pole should be checked against M_{star} , and the dependable composite bending capacity (if considered) of the concrete encased pole should be checked against M_{star2} .

Pole Cantilever Wall Bending (earthquake)

- $KN := 1000 \cdot N$
- $H_w := 2.5 \cdot m$ Height of wall
- $\gamma_a := 18 \cdot \frac{KN}{m^3}$ Soil unit weight
- $K_{aeh} := 0.427$ from M-O equations
- $K_p := 1$ for $\phi = 0$ soil
- $S_u := 50 \cdot \frac{KN}{m^2}$ Undrained shear strength of founding soil
- $\omega := 4 \cdot \frac{KN}{m^2}$ surcharge (factored)
- $\alpha := 1$ load factor, earthquake case
- $L_s := 1.5 \cdot m$ pole spacing

Bending moment in pole at ground level

$$M_{star} := \alpha \cdot K_{aeh} \cdot \left(\frac{1}{2} \omega \cdot H_w^2 + \frac{1}{6} \gamma_a \cdot H_w^3 \right) \cdot L_s \quad M_{star} = 38.03 \cdot KN \cdot m$$

Depth below ground to zero shear (max bending moment)

$$z_{bmax} := 0.94 \cdot m \quad \text{set SF to zero by trial and error}$$

$$SF_{bmax} := \left[K_{aeh} \cdot \left[\omega \cdot (H_w + z_{bmax}) + \frac{1}{2} \gamma_a \cdot (H_w + z_{bmax})^2 \right] - K_p \cdot \left(\frac{1}{2} \gamma_a \cdot z_{bmax}^2 \right) - 2 \cdot S_u \cdot (z_{bmax} - 0.5 \cdot m) \right] \cdot L_s$$

$$SF_{bmax} = -0.901 \cdot KN$$

Maximum bending moment below ground

$$M_{star2} := \alpha \cdot \left[K_{aeh} \cdot \left[\frac{1}{2} \omega \cdot (H_w + z_{bmax})^2 + \frac{1}{6} \gamma_a \cdot (H_w + z_{bmax})^3 \right] - K_p \cdot \left(\frac{1}{6} \gamma_a \cdot z_{bmax}^3 \right) - S_u \cdot (z_{bmax} - 0.5 \cdot m)^2 \right] \cdot L_s$$

$$M_{star2} = 75.121 \cdot KN \cdot m$$

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